

**INTEGRALS AND TRIG!!****Exercises 1**

Use the trigonometric identities stated on page 2 to find the following integrals.

$$1. \text{ (a) } \int \cos^2 x \, dx \quad \text{(b) } \int_0^{\pi/2} \cos^2 x \, dx \quad \text{(c) } \int \sin 2x \cos 2x \, dx$$

$$2. \text{ (a) } \int_{\pi/6}^{\pi/3} 2 \cos 5x \cos 3x \, dx \quad \text{(b) } \int (\sin^2 t + \cos^2 t) dt \quad \text{(c) } \int \sin 7t \sin 4t \, dt.$$

**Exercises 2**

$$1. \text{ (a) Find } \int \cos^3 x \, dx \quad \text{(b) } \int \cos^5 x \, dx \quad \text{(c) } \int \sin^5 x \cos^2 x \, dx.$$

2. Evaluate  $\int \sin^2 x \cos^2 x \, dx$  by using the double angle formulae

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

3. Using the double angle formulae twice find  $\int \sin^4 x \cos^2 x \, dx$ .

**Exercises 3**

1. Use the trigonometric substitution indicated to find the given integral.

$$\text{(a) } \int \frac{x^2}{\sqrt{16 - x^2}} dx \quad \text{let } x = 4 \sin \theta \quad \text{(b) } \int \frac{1}{1 + 4x^2} dx \quad \text{let } x = \frac{1}{2} \tan \theta.$$

**Answers****Exercises 1**

1. (a)  $\frac{x}{2} + \frac{1}{4} \sin 2x + c$     (b)  $\frac{\pi}{4}$     (c)  $-\frac{\cos 4x}{8} + c$

2. (a)  $\frac{\sqrt{3}}{8} = 2.165$  (3 d.p.)    (b)  $t + c$

(c)  $\frac{1}{6} \sin 3t - \frac{1}{22} \sin 11t + c$

**Exercises 2**

1. (a)  $\frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \sin x + c$     (b)  $\frac{1}{5} \cos^4 x \sin x + \frac{4}{15} \cos^2 x \sin x + \frac{8}{15} \sin x + c$   
(c)  $-\frac{1}{7} \sin^4 x \cos^3 x - \frac{4}{35} \sin^2 x \cos^3 x - \frac{8}{105} \cos^3 x + c.$

2.  $-\frac{1}{4} \sin x \cos^3 x + \frac{1}{8} \cos x \sin x + \frac{1}{8} x + c.$

3.  $-\frac{1}{6} \sin^3 x \cos^3 x - \frac{1}{8} \sin x \cos^3 x + \frac{1}{16} \cos x \sin x + \frac{1}{16} x + c$

**Exercises 3**

1. (a)  $-\frac{1}{2} x \sqrt{16 - x^2} + 8 \sin^{-1} \frac{x}{4} + c$     (b)  $\frac{1}{2} \tan^{-1} 2x + c.$